# Work Done by Constant and Variable Forces

## Comprehensive Notes for NEET Physics Exam Preparation

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**Topic: Work, Energy and Power - Chapter 6**

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## 1. Introduction to Work

Work is said to be done when a force applied on a body causes displacement in the direction of the force. It is a scalar quantity with dimensions $[ML^2T^{-2}]$ and SI unit Joule (J).

### Key Points

- Work is the dot product of force and displacement vectors
- It can be positive, negative, or zero depending on the angle between force and displacement
- Work is a scalar quantity but derived from vector quantities
- 1 Joule = 1 Newton × 1 meter

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## 2. Work Done by a Constant Force

### 2.1 Definition and Formula

When a constant force $\vec{F}$ acts on a body and causes a displacement $\vec{S}$, the work done is defined as:

$$W = \vec{F} \cdot \vec{S} = FS\cos\theta$$

where $\theta$ is the angle between the force vector and displacement vector.

### 2.2 Special Cases

**Case 1: Force and displacement in same direction** ($\theta = 0^\circ$)

$$W = FS\cos 0^\circ = FS$$

Work is maximum and positive.

**Case 2: Force perpendicular to displacement** ($\theta = 90^\circ$)

$$W = FS\cos 90^\circ = 0$$

No work is done (e.g., centripetal force in uniform circular motion).

**Case 3: Force opposite to displacement** ($\theta = 180^\circ$)

$$W = FS\cos 180^\circ = -FS$$

Work is negative (e.g., friction, air resistance).

### 2.3 Important Concepts

- **Positive Work**: When force aids the motion ($0^\circ \le \theta < 90^\circ$). Energy is added to the system.
- **Negative Work**: When force opposes the motion ($90^\circ < \theta \le 180^\circ$). Energy is removed from the system.
- **Zero Work**: When force is perpendicular to displacement or when displacement is zero.

### 2.4 Work Done by Gravity

When a body of mass $m$ falls freely through a height $h$:

$$W_{\text{gravity}} = mgh$$

When a body is lifted up against gravity through height $h$:

$$W_{\text{against gravity}} = -mgh$$

### 2.5 Work Done in Multiple Dimensions

If a constant force $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ causes displacement $\vec{S} = S_x\hat{i} + S_y\hat{j} + S_z\hat{k}$, then:

$$W = \vec{F} \cdot \vec{S} = F_xS_x + F_yS_y + F_zS_z$$

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## 3. Work Done by a Variable Force

### 3.1 Definition and Concept

When force varies with position, the elementary work done over a small displacement $dx$ is:

$$dW = F(x)dx$$

Total work done from position $x_1$ to $x_2$ is obtained by integration:

$$W = \int_{x_1}^{x_2} F(x)\, dx$$

### 3.2 Graphical Interpretation

The work done by a variable force equals the area under the Force vs. Displacement graph between the given limits.

*[Area under F-x curve represents work done]*
Figure 1: Work done equals area under force-displacement curve

### 3.3 Work Done by Spring Force (Hooke's Law)

For an ideal spring with spring constant $k$, the restoring force is:

$$F = -kx$$

Work done in stretching or compressing a spring from natural length by distance $x$ is:

$$W = \int_0^x kx\,dx = \frac{1}{2}kx^2$$

Work done in stretching from $x_1$ to $x_2$:

$$W = \frac{1}{2}k(x_2^2 - x_1^2)$$

### 3.4 General Variable Force Examples

**Example 1: Exponential Force**
If $F(x) = F_0 e^{-kx}$, work done from $x = 0$ to $x = a$:

$$W = \int_0^a F_0 e^{-kx}\,dx = \frac{F_0}{k}(1 - e^{-ka})$$

**Example 2: Power Law Force**
If $F(x) = ax^n$, work done from $x_1$ to $x_2$:

$$W = \int_{x_1}^{x_2} ax^n\,dx = \frac{a}{n+1}(x_2^{n+1} - x_1^{n+1})$$

### 3.5 Work Done by Time-Dependent Forces

When force depends on time and velocity, work can be calculated as:

$$W = \int_{t_1}^{t_2} \vec{F}(t) \cdot \vec{v}(t)\,dt$$

where $\vec{v}(t)$ is the velocity at time $t$.

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## 4. Important Formulas Summary

| Concept | Formula |
| --- | --- |
| Work by constant force | $W = FS\cos\theta$ |
| Work by variable force | $W = \int_{x_1}^{x_2} F(x)\,dx$ |
| Work by gravity (downward) | $W = mgh$ |
| Work by spring force | $W = \frac{1}{2}k(x_2^2 - x_1^2)$ |
| Work in vector form | $W = \vec{F}\cdot\vec{S}$ |
| Power | $P = \frac{dW}{dt} = \vec{F}\cdot\vec{v}$ |

Table 1: Quick reference formulas for work done

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# 5. Solved MCQs with Detailed Explanations

## MCQ Set 1: Work Done by Constant Force

**Q1. A force of 10 N acts on a body of mass 2 kg for 3 seconds. The body was initially at rest. What is the work done by the force?**

(A) 30 J
(B) 45 J
(C) 90 J
(D) 225 J

**Answer: (D) 225 J**

**Explanation:**

- Acceleration: $a = \frac{F}{m} = \frac{10}{2} = 5\ \text{m/s}^2$
- Displacement in 3 seconds: $s = \frac{1}{2}at^2 = \frac{1}{2}\times 5\times 9 = 22.5\ \text{m}$
- Work done: $W = F\times s = 10\times 22.5 = 225\ \text{J}$

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**Q2. A block of mass 5 kg is pulled along a horizontal surface by a force of 20 N making an angle of 60° with the horizontal. If the block moves 10 m, what is the work done?**

(A) 100 J
(B) 173 J
(C) 200 J
(D) 346 J

**Answer: (A) 100 J**

**Explanation:**

- $W = FS\cos\theta = 20\times 10\times\cos 60^\circ$
- $W = 20\times 10\times 0.5 = 100\ \text{J}$

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**Q3. A porter lifts a luggage of 20 kg from the ground and places it on his head (1.5 m above the ground). Calculate the work done by him on the luggage. ($g = 10\ \text{m/s}^2$)**

(A) 200 J
(B) 250 J
(C) 300 J
(D) 350 J

**Answer: (C) 300 J**

**Explanation:**

- Work done against gravity: $W = mgh$
- $W = 20 \times 10 \times 1.5 = 300$ J

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**Q4. A body is moved along a straight line by a machine delivering constant power. The distance moved by the body in time t is proportional to:**

(A) $t^{1/2}$
(B) $t^{3/4}$
(C) $t^{3/2}$
(D) $t^2$

**Answer: (C) $t^{3/2}$**

**Explanation:**

- Power $P = \frac{dW}{dt} = F \cdot v = ma \cdot v$
- For constant power: $P = m\frac{dv}{dt} \cdot v$
- Solving this differential equation gives $v \propto t^{1/2}$
- Since $v = \frac{ds}{dt}$, integrating gives $s \propto t^{3/2}$

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**Q5. A force $\vec{F} = (3\hat{i} + 4\hat{j})$ N acts on a particle and produces a displacement $\vec{S} = (2\hat{i} - 3\hat{j})$ m. The work done is:**

(A) -6 J
(B) 0 J
(C) 6 J
(D) 18 J

**Answer: (A) -6 J**

**Explanation:**

- $W = \vec{F} \cdot \vec{S} = (3)(2) + (4)(-3)$
- $W = 6 - 12 = -6$ J

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## MCQ Set 2: Work Done by Variable Force

**Q6. The force acting on a particle varies as shown in the graph. The work done by the force in displacing the particle from x = 0 to x = 10 m is:**

[For a triangular graph with F = 10 N at x = 0, decreasing linearly to F = 0 at x = 10 m]

(A) 25 J
(B) 50 J

(C) 75 J
(D) 100 J

**Answer: (B) 50 J**

**Explanation:**

- Work = Area under F-x curve
- Area of triangle = $\frac{1}{2} \times \text{base} \times \text{height}$
- $W = \frac{1}{2} \times 10 \times 10 = 50 \text{ J}$

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**Q7. A spring of spring constant k = 500 N/m is compressed by 10 cm. The work done in compressing the spring is:**

(A) 0.5 J
(B) 2.5 J
(C) 5 J
(D) 25 J

**Answer: (B) 2.5 J**

**Explanation:**

- $W = \frac{1}{2}kx^2$
- $x = 10 \text{ cm} = 0.1 \text{ m}$
- $W = \frac{1}{2} \times 500 \times (0.1)^2 = 2.5 \text{ J}$

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**Q8. A particle is acted upon by a force F which varies with position x as $F = kx^2$ where k is a constant. The work done in displacing the particle from x = 1 m to x = 2 m is:**

(A) $\frac{7k}{3}$
(B) $\frac{5k}{3}$
(C) $3k$
(D) $7k$

**Answer: (A) $\frac{7k}{3}$**

**Explanation:**

- $W = \int_1^2 kx^2 \, dx$
- $W = k\left[\frac{x^3}{3}\right]_1^2 = k\left(\frac{8}{3} - \frac{1}{3}\right)$
- $W = \frac{7k}{3}$

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**Q9. A force $F = (10 + 0.50x)$ acts on a particle in the x-direction, where F is in newtons and x in meters. The work done by this force during displacement from x = 0 to x = 2 m is:**

(A) 20 J
(B) 21 J
(C) 22 J
(D) 41 J

**Answer: (B) 21 J**

**Explanation:**

- $W = \int_0^2 (10 + 0.50x)\,dx$
- $W = \left[10x + 0.50 \times \frac{x^2}{2}\right]_0^2$
- $W = (20 + 0.25 \times 4) - 0 = 20 + 1 = 21 \text{ J}$

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**Q10. The force on a particle as a function of position is given by $F = -kx^3$. The work done by this force in displacing the particle from x = a to x = b (where b > a) is:**

(A) $\frac{k}{4}(b^4 - a^4)$
(B) $-\frac{k}{4}(b^4 - a^4)$
(C) $\frac{k}{3}(b^3 - a^3)$
(D) $-\frac{k}{3}(b^3 - a^3)$

**Answer: (B)** $-\frac{k}{4}(b^4 - a^4)$

**Explanation:**

- $W = \int_a^b (-kx^3)\,dx$
- $W = -k\left[\frac{x^4}{4}\right]_a^b$
- $W = -\frac{k}{4}(b^4 - a^4)$

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## 6. Additional Practice Questions

**Q11. A man pushes a wall with a force of 100 N for 30 minutes. The work done is:**

(A) 3000 J
(B) 180000 J
(C) 0 J
(D) Cannot be determined

**Answer: (C) 0 J**
**Explanation:** No displacement occurs, so $W = F \times 0 = 0$

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**Q12. If the unit of force and length are increased four times, the unit of energy will increase by:**

(A) 4 times
(B) 8 times
(C) 12 times
(D) 16 times

**Answer: (D) 16 times**
**Explanation:** Energy = Force × Length, so $E' = (4F)(4L) = 16FL$

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**Q13. A particle moves in a circle with constant speed. The work done by the centripetal force is:**

(A) Maximum
(B) Minimum
(C) Zero
(D) Depends on radius

**Answer: (C) Zero**
**Explanation:** Centripetal force is always perpendicular to velocity, so $\theta = 90^\circ$ and $W = 0$

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**Q14. A spring is stretched by 5 cm by a force of 10 N. The work done in stretching it further by 5 cm is:**

(A) 0.25 J
(B) 0.50 J
(C) 0.75 J
(D) 1.00 J

**Answer: (C) 0.75 J**
**Explanation:**

- $k = \frac{F}{x} = \frac{10}{0.05} = 200 \text{ N/m}$
- $W = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2} \times 200 \times (0.1^2 - 0.05^2)$
- $W = 100 \times (0.01 - 0.0025) = 0.75 \text{ J}$

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**Q15. A force acting on a body of mass 2 kg varies with position as $F = 2x$ (in SI units). If the body starts from rest at x = 0, its velocity at x = 4 m will be:**

(A) 4 m/s
(B) 8 m/s
(C) 16 m/s
(D) 32 m/s

**Answer: (A) 4 m/s**
**Explanation:**

- Work done: $W = \int_0^4 2x \, dx = \left[x^2\right]_0^4 = 16 \text{ J}$
- By work-energy theorem: $W = \frac{1}{2}mv^2$
- $16 = \frac{1}{2} \times 2 \times v^2 \Rightarrow v = 4 \text{ m/s}$

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# 7. Key Concepts and Common Mistakes

## Important Points to Remember

- Work is a scalar quantity but can be positive, negative, or zero
- Zero displacement always means zero work, regardless of force magnitude
- Perpendicular forces do no work (centripetal force, normal force on horizontal motion)
- For variable forces, always use integration - simple multiplication doesn't apply
- Spring work depends on both initial and final positions, not just displacement
- In circular motion at constant speed, net work by all forces is zero
- Graphical method: area under F-x curve gives work (be careful with signs)

### Common Mistakes to Avoid

- Confusing force with work (force is a vector, work is a scalar)
- Forgetting the $\cos\theta$ factor in work formula
- Using $W = Fs$ for variable forces without integration
- Ignoring the sign of work (positive vs negative)
- Calculating displacement as distance traveled (displacement is vector quantity)
- Not converting units properly (cm to m, N to kN, etc.)
- Forgetting that work by conservative forces is path-independent

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## 8. Quick Revision Points

| Situation | Work Done |
|---|---|
| Lifting object up | Positive (against gravity) |
| Object falling down | Negative (by gravity) |
| Friction always | Negative (opposes motion) |
| Normal force on horizontal surface | Zero (perpendicular) |
| Centripetal force in circular motion | Zero (perpendicular) |
| Stretching/compressing spring | Always positive |

Table 2: Work done in various situations

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## 9. Exam Strategy Tips

1. Always identify whether force is constant or variable before choosing formula
2. Draw free body diagrams to identify all forces acting
3. Check units carefully - convert everything to SI units
4. For integration problems, identify limits correctly
5. Use work-energy theorem to connect work with kinetic energy changes
6. Practice graphical problems - area calculation is frequently tested
7. Remember special cases: spring force, gravitational force, friction
8. Time management: variable force problems take longer - attempt constant force questions first

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## 10. Connection to Other Topics

### Work-Energy Theorem

$$W_{\text{net}} = \Delta KE = \frac{1}{2}m(v_f^2 - v_i^2)$$

### Power

$$P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$$

Average power: $P_{\text{avg}} = \frac{W}{t}$

### Conservative Forces

For conservative forces (gravity, spring, electrostatic):

- Work done is path-independent
- Work in a closed loop is zero
- Can define potential energy

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## Summary

This comprehensive guide covers all essential concepts of work done by constant and variable forces. Master the fundamental formulas, practice the solved MCQs thoroughly, understand the conceptual differences between constant and variable force scenarios, and apply integration techniques where required.

**For NEET Success:**

- Solve at least 50-100 MCQs on this topic
- Understand each solution conceptually, not just memorize
- Practice graphical problems extensively
- Connect work with energy and power concepts
- Time yourself while solving - aim for 1-2 minutes per MCQ

**All the best for your NEET preparation!**

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