MOTION IN A VERTICAL CIRCLE — NEET PHYSICS NOTES

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1. Complete Physical Theory of Speed Changes in a Vertical Circle

In a **horizontal circle**, speed can stay constant because gravity does NOT affect the speed along the path.
But in a **vertical circle**, gravity acts downward, and its effect changes at every point, so speed cannot remain constant.

- Gravity opposes motion when the body moves upward → body slows.
- Gravity helps motion when the body moves downward → body speeds up.

Thus:

- Speed is maximum at the lowest point and minimum at the highest point.
- Speed changes continuously because gravitational potential energy changes continuously.

Energy View: The Best Way to Understand Speed Changes

There are only two energies at play:

- **Kinetic Energy (KE):** depends on speed
- **Gravitational Potential Energy (PE):** depends on height

As the object moves:

- When height increases → PE increases → KE decreases → speed decreases.
- When height decreases → PE decreases → KE increases → speed increases.

Total mechanical energy remains constant (if no friction).

Step by Step: Motion Through the Circle

1. **Bottom of the Circle — Lowest Point**:
   Height is minimum, PE is minimum, KE is maximum, speed is highest. Gravity accelerates the body downward, increasing speed.
2. **Moving Upward (Right or Left Side)**:
   Height increases, PE increases, KE decreases, speed decreases. Gravity opposes motion, slowing the body.
3. **Highest Point — Top of the Circle**:
   Height is maximum, PE is maximum, KE is minimum, speed is lowest. If speed becomes too low, the body can't maintain the circular path (tension falls to zero).
4. **Coming Downward (Top → Bottom)**:
   Height decreases, PE decreases, KE increases, speed increases. Gravity helps accelerate the body downward.

Summary:

- Speed is minimum at the top.
- Speed is maximum at the bottom.
- Speed decreases while going up.
- Speed increases while coming down.
- Speed changes continuously because height (and therefore PE) keeps changing.

One-Line Summary:
Speed changes in a vertical circle because gravitational potential energy changes with height, and total mechanical energy stays constant, so kinetic energy and speed adjust automatically.

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2. Detailed Conceptual Theory of Vertical Circle

Imagine a body attached to a string, moving in a full circle in a vertical plane—up, down, sideways—like a stone tied to a rope and whirled above your head. This is vertical circle motion.

What's special about vertical circle motion?

- Gravity acts downward at all points.
- Speed fluctuates.
- Tension keeps changing.
- There are critical points where the motion may break.

Key Difference from Horizontal Circle:

- In a horizontal circle, speed can remain constant.
- In a vertical circle, gravity assists and opposes motion at different segments—making it more complex.

Why Speed Changes in a Vertical Circle

- **Upward Journey:** Gravity opposes motion, so speed decreases.
- **Downward Journey:** Gravity helps motion, so speed increases.
- **Lowest Point:** PE is minimal, KE (speed) is maximum.
- **Highest Point:** PE is maximal, KE (speed) is minimum.

Energy Conservation

$$E_{\text{mech}} = KE + PE = \text{constant (in absence of friction)}$$

Tension as a Variable Force

Tension has two roles:

1. Keeps the body moving in a circle (centripetal force).
2. Supports/controls the body against gravity at different points.

- Tension is greatest at the lowest point (maximum speed).
- Tension is smallest at the highest point.

If tension drops to zero at the highest point, the string goes slack, breaking the circular motion.

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3. Key Concepts

- Forces at play: Gravity (mg, downward), tension (T, towards centre).
- Energy changes: KE and PE transform at various positions.
- **Critical Velocity:** Minimum velocity at the top to keep the string taut, $v_{\text{top,min}} = \sqrt{gR}$.
- **Mechanical Energy Conservation:** In absence of friction. KE + PE is conserved.

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4. Application to NEET Problems

- Use energy conservation between positions to find unknown velocities.
- Use centripetal force balance to find tension at specific points.

Example:
A stone whirled in a vertical circle of radius R. Minimum speed at the lowest point for a complete circle:

$$v_{\text{min, bottom}} = \sqrt{5gR}$$

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5. Role of Tension at Different Points in the Circle

Lowest Point (Bottom):

$$T_{\text{bottom}} = m\frac{v_{\text{bottom}}^2}{R} + mg$$

- Tension supports both centripetal and weight.

- Body is fastest.
- Most likely break point.

Highest Point (Top):

$$T_{\text{top}} = m\frac{v_{\text{top}}^2}{R} - mg$$

- Tension is minimum.
- Body is slowest.
- Tension must be at least zero for circular motion.

Midway Position:
Resolve forces for gravity and tension components.

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6. Velocities at Different Positions

Energy Conservation (Bottom to Top):

$$KE_{\text{bottom}} + PE_{\text{bottom}} = KE_{\text{top}} + PE_{\text{top}}$$

$$v_{\text{bottom}}^2 = v_{\text{top}}^2 + 4gR$$

Minimum Velocity at Top:

$$v_{\text{top,min}} = \sqrt{gR}$$

(For string to stay taut.)

Minimum Velocity at Bottom for Full Revolution:

$$v_{\text{bottom,min}} = \sqrt{5gR}$$

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7. Important Formulas

- Tension at bottom: $T_{\text{bottom}} = mv_{\text{bottom}}^2/R + mg$
- Tension at top: $T_{\text{top}} = mv_{\text{top}}^2/R - mg$
- Minimum velocity at top: $v_{\text{top,min}} = \sqrt{gR}$
- Minimum velocity at bottom: $v_{\text{bottom,min}} = \sqrt{5gR}$
- Energy conservation: $KE_1 + PE_1 = KE_2 + PE_2$

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8. MCQs (Multiple Choice Questions) with Solutions

1. The tension in the string at the bottom of the vertical circle is:
    - (A) mg
    - (B) $mv^2/R + mg$ **(Correct)**
    - (C) $mv^2/R - mg$
    - (D) mv^2/R
    Solution: At the lowest point, both required centripetal force and weight act in the same direction—tension must provide both.

2. Minimum velocity required at the highest point for revolution is:
   - (A) gR
   - (B) $\sqrt{gR}$ **(Correct)**
   - (C) $2gR$
   - (D) $\sqrt{5gR}$

   Solution: At the top, if tension $= 0$, gravity alone provides centripetal force: $mg = mv^2/R \implies v_{\text{top, min}} = \sqrt{gR}$.
3. In vertical circle motion, speed is maximum at:
   - (A) Top
   - (B) Bottom **(Correct)**
   - (C) Midway
   - (D) All points

   Solution: Gravity helps descent to the bottom, ensuring max speed there.
4. Tension at the top becomes zero—what happens?
   - (A) Body speeds up
   - (B) String tightens
   - (C) Circular path breaks **(Correct)**
   - (D) String lengthens

   Solution: If tension is zero, the string is slack and the mass cannot complete the circle.
5. Difference in tension at bottom and top for a particle moving in a vertical circle is:
   - (A) $2mg$ **(Correct)**
   - (B) mg
   - (C) 0
   - (D) $5mg$

   Solution: Use tension formulas and speed difference: result is $2mg$.

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9. Assertion-Reason Questions with Solutions

1. **Assertion:** Speed at the bottom is always greater than at the top.
   Reason: Energy is converted from kinetic to potential while rising.
   Solution: Both true and reason correctly explains assertion.
2. **Assertion:** Tension is minimum at top.
   Reason: Gravity acts downward, reducing needed centripetal force.
   Solution: Both true; gravity helps at the top.
3. **Assertion:** At the highest point, critical velocity ensures string stays taut.
   Reason: If tension at the top is zero, velocity is $\sqrt{gR}$.
   Solution: Both true; reason is correct explanation.
4. **Assertion:** Gravity always opposes tension at bottom.
   Reason: Both gravity and tension act in opposite directions at bottom.
   Solution: Both true.
5. **Assertion:** Velocity at lowest point determines success of vertical circle.
   Reason: Too low velocity may cause string to slack at top.
   Solution: Both true; reason explains assertion.
6. **Assertion:** Tension at sides is always maximum.
   Reason: Tension balances only force in radial direction.
   Solution: Both false.
7. **Assertion:** At any point, sum of tension and gravity gives required centripetal force.
   Reason: Both forces resolve to provide total radial acceleration.

Solution: Both true.

8. **Assertion:** Mechanical energy is conserved in vertical circle.
Reason: With no friction/air resistance, only gravity and tension act.
Solution: Both true.
9. **Assertion:** Heavier mass requires more tension to maintain vertical circle.
Reason: Centripetal force is proportional to mass.
Solution: Both true.
10. **Assertion:** If body stops at top, it will complete circle.
Reason: Zero velocity at top means tension is maximum.
Solution: Both false.

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10. Key Points to Memorise

- Minimum velocity at top for full circle: $\sqrt{gR}$
- Minimum velocity at bottom for full circle: $\sqrt{5gR}$
- Tension is greatest at bottom, smallest at top
- Speed changes; energy is exchanged between KE and PE
- Gravity opposes upwards, helps downwards motion
- Always check tension at critical positions

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