POTENTIAL ENERGY OF SPRING & CONSERVATION OF MECHANICAL ENERGY

Work, Power & Energy - Chapter Notes

CrackNeet Physics - NEET Physics Notes

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1. Potential Energy of a Spring

1.1 What is a Spring Trying to Do?

Think of an ideal spring:

- It has a **natural length** (its relaxed length when nobody is pulling or pushing it)
- If you **stretch** it, it "wants" to come back
- If you **compress** it (push it shorter), it also "wants" to come back to its natural length

Key Understanding: A spring always tries to restore itself to its natural length. That's why the force is called a **restoring force**.

1.2 Hooke's Law – How Spring Force Behaves

For small stretches or compressions (within elastic limit), the spring force is:

- **Directly proportional** to how much you stretch/compress it
- **Always acts opposite** to the direction of stretch or compression

Examples:

- If you stretch it to the right, the spring pulls to the left
- If you compress it to the left, the spring pushes to the right
- The more you deform it (larger |x|), the stronger the restoring force

Mathematical Expression (Hooke's Law):

$$F = -kx$$

Where:

- F = Restoring force (N)
- k = Spring constant (N/m) - measure of stiffness
- x = Displacement from natural length (m)
- Negative sign indicates force opposes displacement

1.3 Doing Work on a Spring

Imagine you pull a spring slowly from its natural length to some extension x.

What happens:

- At first, the spring force is very small (almost zero at $x = 0$)
- As you keep stretching, the force becomes bigger and bigger
- To stretch it, you must pull **against** that spring force
- So you are doing work **on** the spring

Key Idea: The work you do in stretching or compressing the spring is **stored inside the spring as elastic potential energy**.

Analogy: This is similar to raising a body in a gravitational field:

- You do work against gravity → energy is stored as gravitational potential energy
- You do work against the spring's restoring force → energy is stored as spring potential energy

1.4 Why the Energy Expression Depends on x^2

Because the spring force **increases with** x (not constant), the work done is not "force × distance" with a single value of force.

Conceptual Understanding:

- At small x, force is small, so work added is small
- At large x, force is larger, so each additional bit of stretch adds more work

Graphical Interpretation:

If you plotted spring force vs extension (a straight line through origin), the area under the graph from 0 to x represents the work done on the spring.

- That area is a **triangle**, not a rectangle
- Base: from 0 to x
- Height: from 0 to maximum spring force at x
- Area of triangle = $\frac{1}{2} \times \text{base} \times \text{height}$, and height $\propto x$

Important Concept:

Elastic potential energy increases with the square of extension/compression.

If you double the extension, energy becomes **four times**.

1.5 Where is This Energy "Stored"?

Inside the spring:

On a microscopic level, atoms and molecules are pulled slightly apart or pushed closer, and internal forces between them store energy.

When you release the spring, this stored energy is converted back into:

- Kinetic energy (motion)
- Heat
- Sound
- Other forms, depending on the situation

Definition: Spring potential energy is a way of saying: *"The configuration of the spring is such that it can give back energy if we let it go."*

1.6 Spring in Horizontal vs Vertical Direction

Horizontal Spring (Frictionless Surface)

Setup:

- A block attached to a spring on a smooth horizontal table
- Natural length at the center
- Pull the block, release it

Motion: The spring keeps pulling it back and forth → **simple harmonic motion**

Energy Distribution:

At the extremes:

- Extension/compression **maximum**
- Speed **zero**
- All energy is **potential** (in the spring)

At the middle (natural length):

- Extension = 0
- Spring potential energy = 0
- Speed **maximum**
- All energy is **kinetic**

Energy Transformation:

Energy keeps shuttling between:

- Spring potential energy, and
- Kinetic energy of the block

But the **total mechanical energy stays constant** (if no friction).

Vertical Spring (Mass Hanging)

Setup:

- A mass hanging on a vertical spring
- Both gravity and spring force acting
- At new equilibrium (after it settles), spring is stretched by some amount
- If you pull it down further and release, mass oscillates up and down

Energy Distribution:

In this motion, energy keeps moving between:

- Gravitational potential energy (height)
- Spring potential energy (extension)
- Kinetic energy (speed)

Key Point: Total mechanical energy remains constant if no air resistance or damping.

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2. Conservation of Mechanical Energy (with Spring)

2.1 What is Mechanical Energy?

$$\text{Mechanical Energy} = \text{Kinetic Energy} + \text{Potential Energy}$$

Potential energy can be:

- Gravitational (due to height)
- Elastic (due to spring stretch/compression)
- Other forms if any

Conservation Principle:

When only **conservative forces** act (like gravity and ideal spring force), the total mechanical energy of the system **stays constant**.

This means: Energy doesn't disappear; it just changes form between kinetic and potential (gravitational or spring).

2.2 Spring + Block on Frictionless Surface

Consider: A block attached to a spring on a smooth (frictionless) table.

Initial State:

- You pull the block to the right and hold it
- Spring is stretched → maximum spring potential energy
- Block at rest → zero kinetic energy

Now release the block:

Initially:

- Spring: maximum potential energy (fully stretched)
- Block: zero kinetic energy (momentarily at rest)

As it moves toward the center:

- Spring gets shorter → its potential energy decreases
- Block speeds up → kinetic energy increases

At the center (natural length):

- Spring has zero potential energy (no stretch, no compression)
- Block has maximum kinetic energy (maximum speed)

Beyond the center, towards the other side (compression):

- Spring starts getting compressed → spring potential energy increases again
- Block slows down → kinetic energy decreases

At extreme compression:

- Block again momentarily comes to rest
- Spring potential energy is maximum (but now due to compression, not stretch)

Throughout this motion:

The sum of (kinetic energy + spring potential energy) stays the **same**.

Only the **distribution changes**: sometimes more kinetic, sometimes more potential.

That's conservation of mechanical energy in a pure spring-block system.

2.3 Spring + Gravity (Vertical Spring–Mass System)

Now take a vertical spring with a mass attached.

Important: Here we have **two** potential energies:

- Gravitational potential energy (depends on height)
- Spring potential energy (depends on extension)

Plus we have **kinetic energy** (depends on speed).

When the mass oscillates up and down:

At the highest point:

- Height is greatest → gravitational potential energy is relatively high
- Spring might be less stretched → spring potential energy may be smaller
- Speed is momentarily zero → kinetic energy is zero

As it moves downward:

- Height decreases → gravitational potential energy decreases
- Speed increases → kinetic energy increases
- Spring gets more stretched → spring potential energy increases

At the lowest point:

- Height is minimum → gravitational potential is lowest
- Spring is most stretched → spring potential energy is highest
- Speed again becomes zero for a moment → kinetic energy zero

Energy Conservation:

Even though each part (K, spring PE, gravitational PE) individually changes, the **total mechanical energy**:

$$\text{Kinetic} + \text{Gravitational potential} + \text{Spring potential} = \text{Constant}$$

Provided there is:

- No air resistance
- No friction
- No internal damping in the spring (we assume ideal spring)

Beautiful Example: Energy transformation:

$$\text{Gravitational PE} \leftrightarrow \text{Spring PE} \leftrightarrow \text{Kinetic energy}$$

But total mechanical energy remains the **same**.

2.4 What if Friction or Air Resistance is Present?

If friction or air resistance is present:

- The system **loses** mechanical energy over time
- Energy is not destroyed, but is converted into heat, sound, etc.
- So mechanical energy (K + U) decreases, but total energy of the universe remains constant

Example with Spring + Friction:

- A block attached to a spring on a rough table
- When released, it oscillates, but each swing is smaller than the previous one
- **Why?** Because some of the mechanical energy is lost to heat due to friction in every cycle
- Eventually, the block stops, the spring returns to its natural length, and mechanical energy is almost zero (all converted to heat in surroundings)

So: Conservation of mechanical energy applies strictly only when **non-conservative forces** (like friction) are **absent**.

2.5 Intuitive Summary for Students

- Spring potential energy is energy stored due to stretching or compressing a spring
- The more you stretch/compress (within limits), the more energy is stored (and it increases very fast because it depends on the **square** of extension)
- A spring system constantly exchanges energy between:
  - Spring potential energy
  - Kinetic energy
  - (and sometimes gravitational potential energy)
- In ideal conditions (no friction, no air resistance), the total mechanical energy **stays constant** - only its forms change
- In real systems with friction/air resistance, some mechanical energy is continuously converted to other forms like heat, so the mechanical energy **decreases** with time

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3. Mathematical Theory and Key Formulas

3.1 Hooke's Law

Formula:

$$F = -kx$$

Where:

- F = Restoring force (Newton, N)
- k = Spring constant (N/m) - measures stiffness of spring
- x = Displacement from equilibrium position (meter, m)
- Negative sign indicates force is opposite to displacement

Meaning: The restoring force is directly proportional to displacement and always acts to restore the spring to its natural length.

3.2 Elastic Potential Energy of Spring

Formula:

$$U_{spring} = \frac{1}{2}kx^2$$

Where:

- U_{spring} = Elastic potential energy (Joule, J)
- k = Spring constant (N/m)
- x = Displacement from natural length (m)

Meaning: Energy stored in a spring is proportional to the square of displacement. Doubling displacement quadruples stored energy.

Derivation (Brief):

Work done on spring = Area under F-x graph = $\frac{1}{2} \times kx \times x = \frac{1}{2}kx^2$

This work is stored as potential energy.

3.3 Conservation of Mechanical Energy

For spring-mass system (horizontal, frictionless):

$$E_{total} = KE + U_{spring} = \text{constant}$$

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant}$$

For vertical spring-mass system:

$$E_{total} = KE + U_{gravitational} + U_{spring} = \text{constant}$$

$$\frac{1}{2}mv^2 + mgh + \frac{1}{2}kx^2 = \text{constant}$$

Where:

- m = mass (kg)
- v = velocity (m/s)
- g = acceleration due to gravity (9.8 m/s² or 10 m/s²)
- h = height from reference level (m)
- k = spring constant (N/m)
- x = displacement from natural length (m)

3.4 Maximum Extension/Compression

For a block of mass m attached to spring with initial velocity v_0 at natural length:

Maximum compression/extension occurs when velocity becomes zero:

$$\frac{1}{2}mv_0^2 = \frac{1}{2}kx_{max}^2$$

$$x_{max} = v_0\sqrt{\frac{m}{k}}$$

3.5 Velocity at Any Position

For horizontal spring-block system:

$$v = \sqrt{\frac{k}{m}(A^2 - x^2)}$$

Where A = amplitude (maximum displacement)

Maximum velocity (at equilibrium, $x = 0$):

$$v_{max} = A\sqrt{\frac{k}{m}}$$

3.6 Spring Constant and Stiffness

Spring constant k represents stiffness:

- **Large** k → stiff spring (hard to stretch/compress)
- **Small** k → soft spring (easy to stretch/compress)

Units: N/m (Newton per meter)

For springs in series:

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

For springs in parallel:

$$k_{eq} = k_1 + k_2$$

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4. Key Formulas Summary

Concept	Formula	Meaning
Hooke's Law	$F = -kx$	Restoring force proportional to displacement
Spring PE	$U = \frac{1}{2}kx^2$	Energy stored in stretched/compressed spring
Total ME (horizontal)	$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = E$	KE + Spring PE = constant
Total ME (vertical)	$\frac{1}{2}mv^2 + mgh + \frac{1}{2}kx^2 = E$	KE + GPE + Spring PE = constant
Max extension	$x_{max} = v_0\sqrt{\frac{m}{k}}$	Extension when KE converts to PE
Velocity at position	$v = \sqrt{\frac{k}{m}(A^2 - x^2)}$	Speed at any displacement
Max velocity	$v_{max} = A\sqrt{\frac{k}{m}}$	Speed at equilibrium position

Table 1: Summary of Spring and Energy Formulas

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5. Multiple Choice Questions (MCQs)

Q1. A spring of spring constant 200 N/m is compressed by 0.1 m. The potential energy stored in the spring is:

(A) 1 J
(B) 2 J
(C) 10 J
(D) 20 J

Answer: (A) 1 J

Solution: $U = \frac{1}{2}kx^2 = \frac{1}{2} \times 200 \times (0.1)^2 = \frac{1}{2} \times 200 \times 0.01 = 1$ J

Q2. A block of mass 2 kg attached to a spring (k = 200 N/m) is compressed by 0.2 m and released. The maximum kinetic energy of the block is:

(A) 2 J
(B) 4 J
(C) 8 J
(D) 16 J

Answer: (B) 4 J

Solution: By conservation of energy, maximum KE = Initial PE
$KE_{max} = \frac{1}{2}kx^2 = \frac{1}{2} \times 200 \times (0.2)^2 = 100 \times 0.04 = 4$ J

Q3. If the extension in a spring is doubled, the potential energy stored becomes:

(A) Double
(B) Half
(C) Four times
(D) Remains same

Answer: (C) Four times

Solution: $U \propto x^2$, so if $x \to 2x$, then $U \to 4U$

Q4. A spring of spring constant k is cut into two equal halves. The spring constant of each half is:

(A) k
(B) 2k
(C) k/2
(D) 4k

Answer: (B) 2k

Solution: Spring constant is inversely proportional to length. When length is halved, spring constant doubles.

Q5. The potential energy of a spring is maximum when:

(A) It is at natural length
(B) It is at maximum extension or compression
(C) Velocity is maximum
(D) Force is minimum

Answer: (B) It is at maximum extension or compression

Solution: $U = \frac{1}{2}kx^2$ is maximum when x is maximum (at extreme positions).

Q6. A block attached to a spring oscillates on a frictionless horizontal surface. At the equilibrium position:

(A) Kinetic energy is maximum, potential energy is zero
(B) Kinetic energy is zero, potential energy is maximum
(C) Both KE and PE are maximum
(D) Both KE and PE are zero

Answer: (A) Kinetic energy is maximum, potential energy is zero

Solution: At equilibrium ($x = 0$), spring PE = 0 and velocity is maximum, so KE is maximum.

Q7. A spring of natural length 20 cm is stretched to 25 cm by a force of 10 N. The spring constant is:

(A) 50 N/m
(B) 100 N/m
(C) 200 N/m
(D) 500 N/m

Answer: (C) 200 N/m

Solution: Extension $x = 25 - 20 = 5$ cm = 0.05 m
$k = \frac{F}{x} = \frac{10}{0.05} = 200$ N/m

Q8. A mass of 0.5 kg attached to a spring (k = 50 N/m) is displaced by 0.2 m and released. The maximum velocity is:

(A) 1 m/s
(B) 2 m/s
(C) 3 m/s
(D) 4 m/s

Answer: (B) 2 m/s

Solution: $v_{max} = A\sqrt{\frac{k}{m}} = 0.2\sqrt{\frac{50}{0.5}} = 0.2\sqrt{100} = 0.2 \times 10 = 2$ m/s

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Q9. The mechanical energy of a spring-mass system undergoing SHM is:

(A) Conserved only at extreme positions
(B) Conserved only at mean position
(C) Conserved at all positions
(D) Not conserved

Answer: (C) Conserved at all positions

Solution: In absence of friction and air resistance, total mechanical energy is conserved at all positions.

Q10. A block of mass 1 kg falls from height 5 m onto a spring (k = 1000 N/m) placed vertically. Taking g = 10 m/s^2, the maximum compression of spring is approximately:

(A) 0.1 m
(B) 0.2 m
(C) 0.3 m
(D) 0.4 m

Answer: (C) 0.3 m

Solution: By energy conservation:
$mgh = \frac{1}{2}kx^2$ (ignoring the additional fall distance during compression for approximation)
$1 \times 10 \times 5 = \frac{1}{2} \times 1000 \times x^2$
$50 = 500x^2$

$x^2 = 0.1$
$x = 0.316 \text{ m} \approx 0.3 \text{ m}$

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6. Assertion and Reasoning Questions

Instructions: Each question contains an Assertion (A) and a Reason (R). Choose:

(a) Both A and R are true, and R is the correct explanation of A
(b) Both A and R are true, but R is NOT the correct explanation of A
(c) A is true, but R is false
(d) A is false, but R is true

Q1.

Assertion (A): Spring force is a conservative force.

Reason (R): Potential energy is defined only for conservative forces.

Answer: (a) Both A and R are true, and R is the correct explanation of A

Explanation: Spring force is indeed conservative because work done depends only on initial and final positions, not path. Potential energy can only be defined for conservative forces.

Q2.

Assertion (A): A spring has potential energy both when it is compressed and when it is stretched.

Reason (R): In compressing or stretching, work is done on the spring against the restoring force.

Answer: (a) Both A and R are true, and R is the correct explanation of A

Explanation: Work is done against restoring force in both cases, storing energy as elastic PE in the spring.

Q3.

Assertion (A): Total work done by spring force may be positive, negative, or zero.

Reason (R): Direction of spring force is always towards mean position.

Answer: (b) Both A and R are true, but R is NOT the correct explanation of A

Explanation: Spring force always acts toward equilibrium (R is true). Work done by spring can be positive (moving toward equilibrium), negative (moving away), or zero over a complete cycle (A is true), but R doesn't directly explain A.

Q4.

Assertion (A): The elastic potential energy increases with the square of extension.

Reason (R): Work done on a spring varies linearly with displacement.

Answer: (c) A is true, but R is false

Explanation: $U = \frac{1}{2}kx^2$ shows PE varies with x^2 (A is true). But work done is area under F-x graph (triangle), which varies as x^2, not linearly (R is false).

Q5.

Assertion (A): Total mechanical energy in an ideal spring-block system remains constant.

Reason (R): No non-conservative forces act in the system.

Answer: (a) Both A and R are true, and R is the correct explanation of A

Explanation: In absence of friction/air resistance (no non-conservative forces), mechanical energy is conserved.

Q6.

Assertion (A): Velocity is zero when spring is at maximum compression.

Reason (R): Kinetic energy is zero at maximum compression.

Answer: (a) Both A and R are true, and R is the correct explanation of A

Explanation: At extreme positions, all energy is potential, so KE = 0, meaning velocity = 0.

Q7.

Assertion (A): Hooke's Law is applicable beyond elastic limit.

Reason (R): The restoring force is always proportional to displacement.

Answer: (d) Both A and R are false

Explanation: Hooke's Law is valid only within elastic limit. Beyond it, spring deforms permanently and force is not proportional to displacement.

Q8.

Assertion (A): When a spring is at its natural length, its potential energy is zero.

Reason (R): Work done on the spring is zero at natural length.

Answer: (a) Both A and R are true, and R is the correct explanation of A

Explanation: At natural length, $x = 0$, so $U = \frac{1}{2}kx^2 = 0$. No work has been done on the spring.

Q9.

Assertion (A): Energy transformation in spring systems involves kinetic, spring potential, and sometimes gravitational potential energy.

Reason (R): Mechanical energy is conserved only in ideal systems.

Answer: (b) Both A and R are true, but R is NOT the correct explanation of A

Explanation: Both statements are true but independent. Energy transformation occurs in all spring systems; conservation occurs only in ideal (frictionless) systems.

Q10.

Assertion (A): The spring constant of a hard spring is more than that of a soft spring.

Reason (R): Spring constant measures the stiffness of the spring.

Answer: (a) Both A and R are true, and R is the correct explanation of A

Explanation: Spring constant k represents stiffness. Hard spring = large k, soft spring = small k.

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7. Common Mistakes Students Make

Mistake 1: Confusing Force with Potential Energy

Error: Thinking spring force and spring potential energy are the same.

Correction:

- Force: $F = -kx$ (linear in x)
- Potential Energy: $U = \frac{1}{2}kx^2$ (quadratic in x)

Mistake 2: Applying Hooke's Law Beyond Elastic Limit

Error: Using $F = -kx$ for large deformations where spring is permanently deformed.

Correction: Hooke's Law is valid only within elastic limit. Beyond it, spring behavior is non-linear.

Mistake 3: Forgetting the Negative Sign in Hooke's Law

Error: Writing $F = kx$ instead of $F = -kx$.

Correction: Negative sign indicates force opposes displacement (restoring nature).

Mistake 4: Mistaking Maximum PE Position for Maximum Velocity

Error: Thinking velocity is maximum when spring is maximally stretched/compressed.

Correction:

- Maximum PE (zero KE, zero velocity) → at extreme positions
- Maximum KE (maximum velocity) → at equilibrium position ($x = 0$)

Mistake 5: Ignoring Energy Loss Due to Friction

Error: Applying conservation of mechanical energy when friction is present.

Correction: With friction, mechanical energy decreases continuously. Use:

$$\Delta KE + \Delta PE = -W_{friction}$$

Mistake 6: Confusing Total ME with Individual Energies

Error: Thinking total mechanical energy varies even in ideal systems.

Correction: In ideal (frictionless) systems, total ME = constant, but individual KE and PE exchange.

Mistake 7: Unit Errors

Error: Mixing units (cm instead of m, grams instead of kg).

Correction: Always convert to SI units: meters, kilograms, seconds before calculation.

Mistake 8: Wrong Application of Energy Conservation in Vertical Springs

Error: Ignoring gravitational PE in vertical spring-mass systems.

Correction: For vertical springs, include all three:

$$E = KE + U_{spring} + U_{gravity}$$

Mistake 9: Assuming Spring Constant Changes with Deformation

Error: Thinking k varies as spring is stretched more.

Correction: Spring constant k is a property of the spring material and geometry—it remains constant (within elastic limit).

Mistake 10: Not Accounting for Equilibrium Position in Vertical Springs

Error: Taking natural length as reference for vertical springs.

Correction: In vertical springs, equilibrium position is where $kx_0 = mg$, not the natural length. Use this as reference for oscillations.

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8. Important Concepts and Tips

Energy Exchange in Spring Systems

- At extreme positions: All PE, zero KE
- At equilibrium: All KE, zero spring PE
- Energy continuously transforms but total remains constant (ideal case)

Spring Combinations

Springs in Series: Effective spring constant decreases

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

Springs in Parallel: Effective spring constant increases

$$k_{eq} = k_1 + k_2$$

Work Done by Spring Force

Work done by spring force in moving from position x_1 to x_2:

$$W = -\frac{1}{2}k(x_2^2 - x_1^2)$$

Negative sign because spring force opposes displacement from equilibrium.

Practice Strategy

1. Always identify whether system is horizontal or vertical
2. Write energy conservation equation appropriate to the system
3. Identify initial and final states clearly
4. Check if friction/air resistance is present
5. Convert all quantities to SI units
6. Verify answer has correct units

Summary

Key Takeaways:

- Spring force is a conservative restoring force: $F = -kx$
- Elastic PE stored in spring: $U = \frac{1}{2}kx^2$ (depends on x^2)
- Total mechanical energy is conserved in ideal spring systems
- Energy transforms between KE, spring PE, and gravitational PE
- At extreme positions: maximum PE, zero KE
- At equilibrium: maximum KE, zero spring PE
- Friction/air resistance causes mechanical energy to decrease
- Hooke's Law valid only within elastic limit

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Master the concepts, understand energy transformations, and practice systematically. All the best for your NEET preparation!
